

Double Symmetries in Field Theories

L.M.Slad ¹

*D.V.Skobeltsyn Institute of Nuclear Physics,
Moscow State University, Moscow 119899*

Abstract

In the paper a concept of a double symmetry is introduced, and its qualitative characteristics and rigorous definitions are given. We describe two ways to construct the double-symmetric field theories and present an example demonstrating the high efficiency of one of them. In noting the existing double-symmetric theories we draw attention at a dual status of the group $SU(2)_L \otimes SU(2)_R$ as a secondary symmetry group, and in this connexion we briefly discuss logically possible aspects of the P -violation in weak interactions.

¹E-mail: slad@theory.npi.msu.su

1. Qualitative characteristics of the double symmetry

Extension of symmetry approaches to the field theories construction is of an ordinary practice in physics. In the present paper we propose some generalization of already existing approaches which consist in constructing a new group \mathcal{G}_T , called a double symmetry group, on the basis of the given global group G (the primary symmetry group) and some its representation T . This construction is carried out in the framework of group transformations in field-vectors space. Namely, the transformations of the double symmetry group consist of transformations of the primary and secondary symmetries, realized in the same field-vectors space of some representation S of the group G . The secondary symmetry transformations, which can be global as well as local, have two features. Firstly, the parameters of these transformations belong to the space of the representation T of the group G . Secondly, the secondary symmetry transformations do not violate the primary symmetry.

As a rule, the secondary symmetry transformations couple different irreducible representations, into which the representations S of the group G is decomposed, removing or reducing the arbitrariness of a field theory allowed by its invariance in respect to G . If one or another secondary symmetry corresponds to the real physical world, then this correspondence is almost inevitably concealed by its spontaneous breaking.

We restrict ourselves with formulations of the global symmetries. The knowledge of the secondary symmetry group allows to introduce the relevant local transformations and gauge fields in the standard way, if there is any need of this.

2. Rigorous definitions of the secondary and double symmetries

Definition 1. Assume that there are a symmetry group G of some field theory and two its representations T and S . Let $\theta = \{\theta_a\}$ be a vector in the representation space of T , $\Psi(x)$ be any field vector in the representation space of S , and let D^a be such operators that the field $\Psi'(x)$, obtained by the transformation

$$\Psi'(x) = \exp(-iD^a\theta_a)\Psi(x), \quad (1)$$

belongs again to the representation space of S , i.e. for any $g \in G$

$$\exp(-iD^b(T(g)\theta)_b)S(g)\Psi(x) = S(g)\Psi'(x). \quad (2)$$

Then the transformations (1) and their products will be called secondary symmetry transformations produced by the representation T of the group G .

The relations (1) and (2) allows to give a more general definition.

Definition 2. Let the group \mathcal{G}_T contain the subgroup G and the invariant subgroup H_T , with $G \neq \mathcal{G}_T$, $H_T \neq \mathcal{G}_T$, and $\mathcal{G}_T = H_T \circ G$. If any element $h_T \in H_T$ can be written in the form

$$h_T = h(\theta_1)h(\theta_2)\dots h(\theta_n), \quad n \in \{1, 2, \dots\}, \quad (3)$$

where θ_i ($i = 1, 2, \dots, n$) is some vector of the representation space of T of the group G , and if for any element $g \in G$

$$gh(\theta)g^{-1} = h(T(g)\theta), \quad (4)$$

then the group G will be called the primary symmetry group, and the groups H_T and \mathcal{G}_T will be respectively called the secondary and double symmetry groups produced by the representation T of the group G .

It follows from the relations (1) and (2) that the operators D^a must satisfy the condition

$$D^a = S^{-1}(g)D^bS(g)[T(g)]_b^a, \quad (5)$$

i.e., as one uses to say, such operators D^a transform as the representation T of the group G .

If G is a Lie group, then within some vicinity U of the unity element e of the group G the transformation operators $S(g)$ and $T(g)$ can be written as

$$S(g) = \exp(-iL^j\epsilon_j), \quad (6)$$

$$T(g) = \exp(-iM^j\epsilon_j), \quad (7)$$

where L^j and M^j are generators of the group G of the representations S and T , $\epsilon_j = \epsilon_j(g)$ are group parameters corresponding to the given element $g \in U$.

By using Eqs. (6) and (7), condition (5) takes the form

$$[L^j, D^a] = D^b(M^j)_b^a. \quad (8)$$

Expanding the exponent of Eq. (1) in a power series, we see that the first term (Ψ) transforms as S and the second term ($\theta \cdot \Psi$) does as a direct product $T \otimes S$. The following statement is getting obvious. Nontrivial ($D^a \neq 0$) secondary symmetry transformations (1) exist if and only if among irreducible representations of the direct product $T \otimes S$ there is at least one which belongs to the representation S . If the operators D^a are matrix ones, then their elements are nothing but the unnormalized Clebsch-Gordon coefficients for relevant irreducible representations of the group G to be found from the relation (8). Namely, let ω_t , ω_p , and ω_q denote three irreducible representations of the group G , such that $\omega_t \in T$, $\omega_p \in S$, and $\omega_q \in T \otimes S$, and let α_t , α_p and α_q be a set of indices that characterize vectors in the spaces of relevant irreducible representations. Then we have the well-known relations

$$D_{\omega_q\alpha_q, \omega_p\alpha_p}^{\omega_t\alpha_t} = 0, \quad \text{if } \omega_q \notin S, \quad (9)$$

$$D_{\omega_q\alpha_q, \omega_p\alpha_p}^{\omega_t\alpha_t} = d_{\omega_q\omega_p}^{\omega_t}(\omega_t\alpha_t\omega_p\alpha_p|\omega_q\alpha_q), \quad \text{if } \omega_q \in S, \quad (10)$$

where $(\omega_t\alpha_t\omega_p\alpha_p|\omega_q\alpha_q)$ are the properly normalized Clebsch-Gordon coefficients, $d_{\omega_q\omega_p}^{\omega_t}$ are arbitrary quantities independent on the indices α_t , α_p and α_q .

3. Two ways to construct double-symmetric field theories

Structure of the groups H_T and \mathcal{G}_T , produced by the representation T of the group G , depends on the representation S of the group G and on numerical values of the quantities determining the operators D^a .

If the secondary symmetry is produced by the adjoint representation of the group G and the operators D^a of Eq. (1) coincide with the group generators, then obviously the group H_T is locally isomorphic to the group G . In such a case we shall say that the double symmetry is degenerated one.

In the general case there are at least two ways to complete the construction of the group \mathcal{G}_T and to construct the double-symmetric field theories, i.e. the theories whose Lagrangians are invariant under transformations of the group \mathcal{G}_T .

The first way is to close the algebra of the operators D^a starting from those or other considerations on the fields in question and their interactions. Possessing the Lie algebra of the group \mathcal{G}_T one can find then its representations and allowed double-symmetric theories. One proceeds so in the supersymmetry theory [1-3].

Describing the second way for some fixed representation T , we consider, for simplicity, the matrix realization of the operators D^a . First we choose some class of representations of the group G which seem to be admissible for the field $\Psi(x)$. Some set of arbitrary quantities $d_{\omega_q\omega_p}^{\omega_t}$ determining the operators D^a through Eq. (10) corresponds to each representation of this class. It is required the Lagrangian of the considered theory to be invariant under transformations of the group G as well as under global transformations (1). Then, obviously, it will be invariant under any transformations of the groups H_T and \mathcal{G}_T . These requirements can lead to some selection of the representations of the considered class, and restrict the arbitrariness of quantities $d_{\omega_q\omega_p}^{\omega_t}$ and the arbitrariness of Lagrangian constants allowed by the invariance in respect to the group G . If the arbitrariness of quantities $d_{\omega_q\omega_p}^{\omega_t}$ is removed completely (up to a common constant) for each of selected representations of the group G , then the structure of the group H_T becomes automatically fully defined, but, generally speaking, it is unlike for different representations of the group G . If it is not so, only then a question arises on closing the algebra of the operators D^a taking into account the obtained restrictions. Thus, on this way the first place is taken by constraints established by the double symmetry produced by the representation T of the group G , and the problem on the Lie algebra of the group \mathcal{G}_T is resolved automatically or moved onto the last place. It seems to be reasonable to demonstrate the efficiency of the described way with the help of a nontrivial example. Let us do this.

4. An example of constructing double-symmetric field theory: the efficiency of selecting representations and removing ambiguities

Consider the relativistically invariant ² Lagrangian of the general type [4] for some free fermionic field $\Psi(x)$

$$\mathcal{L}_0 = \frac{i}{2}[(\partial_\mu \Psi, L^\mu \Psi) - (\Psi, L^\mu \partial_\mu \Psi)] - \kappa(\Psi, \Psi), \quad (11)$$

where (Ψ_1, Ψ_2) is a relativistically invariant bilinear form, L^μ are matrix operators, and κ is a constant.

Require the Lagrangian (11) to be invariant under the global secondary symmetry transformations produced by a polar 4-vector of the group L^\uparrow

$$\Psi'(x) = \exp(-iD^\mu \theta^\mu) \Psi(x), \quad (12)$$

where D^μ are matrix operators. This requirement will be fulfilled if

$$[L^\mu, D^\nu] = 0, \quad (13)$$

$$(D^0 \Psi_1, \Psi_2) = (\Psi_1, D^0 \Psi_2). \quad (14)$$

The condition (5) for the operators D^μ and the condition, which the operators L^μ of the Lagrangian (11) obey, are equivalent. Therefore the matrix elements of both operators L^μ and D^μ are given by relations of the type (9-10). In the monograph [4] there are given their explicit forms. We shall use notations ³ $\tau = (l_0, l_1)$ of Ref. [4] for irreducible

²Here, as well as in the monograph [4], the relativistic invariance means, in modern titles and notations [5], an invariance in respect to the orthochronous Lorentz group L^\uparrow generated by the proper Lorentz group L_+^\uparrow and spatial reflection P .

³In these notations all irreducible representations of the group L_+^\uparrow are elegantly described: finite- and infinite-dimensional ones. Transition to the often used notations (j_1, j_2) , which are connected with the group $SO(4) = SO(3) \otimes SO(3)$ and describe only finite-dimensional irreducible representations, is the following: $j_1 = (l_1 + l_0 - 1)/2$, $j_2 = (l_1 - l_0 - 1)/2$.

representations of the proper Lorentz group and denote the arbitrary quantities of Eq. (10), characterizing the operators L^μ and D^μ , as $c_{\tau'\tau}$ and $d_{\tau'\tau}$, respectively.

Let the representation S of the group L^\uparrow , as which the field $\Psi(x)$ of the Lagrangian (11) transforms, be decomposable into a finite or infinite direct sum of irreducible finite-dimensional representations with half-integer spins, and let the multiplicity of any of these irreducible representations do not exceed 1. Then the Lagrangian (11) will be invariant under global transformations (12) if and only if S is one of the infinite-dimensional representations S^{k_1}

$$S^{k_1} = \sum_{n_1=0}^{\infty} \sum_{k_0=-k_1+1}^{k_1-1} \oplus(k_0, k_1 + n_1), \quad k_1 = 3/2, 5/2, \dots \quad (15)$$

or is the representation S^F containing all irreducible representations of the group L_+^\uparrow with half-integer spins.

For each of these representations all the quantities $|c_{\tau'\tau}|$ and $|d_{\tau'\tau}|$ have, up to their common constants, fully defined values. For any of the representations (15) we get in result

$$[D^\mu, D^\nu] = 0, \quad (16)$$

i.e. the secondary symmetry group H_T is the four-parameter Abelian group. Notice that, although in this case the Lie algebra of the double symmetry group \mathcal{G}_T coincides with the Lie algebra of the Poincaré group, the operators D^μ cannot be identified with the translation operators P^μ because D^μ act only on spin variables of a field but P^μ do not act on them.

The mass spectra corresponding to each of the obtained Lagrangians (11) are infinitely degenerated in spin and continuous. In order to eliminate this degeneration it is necessary to break the secondary symmetry spontaneously keeping the orthochronous Lorentz group symmetry.

5. On a perspective to use the double symmetry

The considered example of the double symmetry, playing here only the methodological role, is used by us as one of elements for constructing a theory with the fields which transform as orthochronous Lorentz group representations decomposable into an infinite direct sum of finite-dimensional irreducible representations⁴. Former the attempts to analyze the theory with such class fields were not undertaken because of the infinite number of arbitrary constants and the absence of criterions for removing this arbitrariness. Carried out sometime ago investigations of the infinite-component field theories, which were assigned to an alternative description of hadrons as composite particles, were based on those representations of the Lorentz group that could be decomposed into a finite direct sum of infinite-dimensional irreducible representations or on a special representation of the group $SO(4, 1)$ or $SO(4, 2)$. It was established, however, that such theories possesses some properties (peculiarities of mass spectra, nonlocality, violation of CPT -invariance and the connection between spin and statistics) which are not admissible for particle physics. Problems, covered by these investigations as well as a sufficiently full list of relevant papers, can be found in the monograph [5].

6. Existing undegenerated double symmetry: supersymmetry

⁴Two our articles on the theory of infinite-component fields with the double symmetry, produced by the polar and axial 4-vectors of the group L^\uparrow , are preparing for publication

Notice now that supersymmetry both in the x -space and in the superspace can be considered as a double symmetry produced by the bispinor representation T of the proper Lorentz group. An element $h(\theta)$ of the corresponding secondary symmetry group H_T has a form [2]

$$h(\theta) = \exp(iQ_\alpha\theta^\alpha + i\overline{Q}_{\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}), \quad (17)$$

the parameters θ^α and $\overline{\theta}^{\dot{\alpha}}$ being anticommutating elements of the Grassmann algebra and belonging to the representation spaces of $(1/2, 0)$ and $(0, 1/2)$ of the proper Lorentz group, respectively (in the notations connected with the group $SO(4)$). A fulfilment of the relation (8), as a criterion of the secondary symmetry, was already required in respect to the bispinor operators Q_α and $\overline{Q}_{\dot{\alpha}}$ in the first paper on the supersymmetry algebra [1]. In the supersymmetry theory the operators Q_α and $\overline{Q}_{\dot{\alpha}}$ are given in the form of sum of terms containing a first or zero power of the differential operator ∂_μ (cf. [2-3]). Such realization of the operators Q_α and $\overline{Q}_{\dot{\alpha}}$ leads to that the secondary symmetry group H_T , generated by the elements (17), contains the space-time translation group, and the double symmetry group \mathcal{G}_T contains the Poincaré group. If one chose a matrix realization for the operators Q_α and $\overline{Q}_{\dot{\alpha}}$, then we would come out of the supersymmetry theory standards.

Let for a given representation S of the group L_+^\uparrow the relation (8) for the spinor operators Q_α in some its realization keep arbitrary N constants. Then there are N linearly independent spinor operators $Q_\alpha^1, Q_\alpha^2, \dots, Q_\alpha^N$, in this realization, satisfying the relation (8). In this case, on one hand, one can introduce a secondary symmetry produced by the N -multiple bispinor representation of the proper Lorentz group, and, on another hand, treat the numbers $1, 2, \dots, N$ of the spinor operators as an index related to some inner symmetry. If as well the operators Q_α^j and $\overline{Q}_{\dot{\alpha}}^j$ ($j = 1, \dots, N$) keep a linear dependence on the operator ∂_μ , then the relevant double symmetry is an extended supersymmetry (cf. [2-3]).

7. Existing undegenerated double symmetry: the σ -model symmetry

Another example of an undegenerated double symmetry, used in particle physics, is the σ -model symmetry, the most accurately described by Gell-Mann and Levy [6]. Infinitesimal transformations in the σ -model have the forms

$$N' = (1 - \frac{i}{2}\gamma^5\boldsymbol{\tau}\boldsymbol{\theta})N, \quad (18)$$

$$\boldsymbol{\pi}' = \boldsymbol{\pi} + i\boldsymbol{\theta}\boldsymbol{\sigma}, \quad (19)$$

$$\sigma' = \sigma - i\boldsymbol{\theta}\boldsymbol{\pi}, \quad (20)$$

where the field N is a nucleon isodoublet, the field $\boldsymbol{\pi}$ is a pseudoscalar isotriplet, the field σ is a scalar isosinglet, and the transformation parameter $\boldsymbol{\theta}$ is a pseudoscalar isotriplet.

The listed transformation properties of the field and the parameter $\boldsymbol{\theta}$ allow to state that the transformations (18) and (19-20) are transformations of the secondary symmetry produced by the representation $T = (\textit{isotriplet}, \textit{pseudoscalar})$ or, that is equivalent, by the representation $T = (\textit{isotriplet}, \textit{scalar}) \oplus (\textit{isotriplet}, \textit{pseudoscalar})$ of the group $G = SU(2) \otimes L^\uparrow$. We wish especially emphasize that, firstly, transformations (18) and (19-20) do not violate the spatial reflection symmetry, and so the group G contains the orthochronous Lorentz group, and, secondly, the parity of fields, involved in the transformations (19-20), are necessarily different due to a pseudoscalar character of the parameter $\boldsymbol{\theta}$. $G_T = SU(2)_L \otimes SU(2)_R$ is a group of the secondary symmetry generated by the transformations (18) and (19-20), the parameters of one of the group $SU(2)$ being

given by sum of the space scalar and pseudoscalar, and the other group parameters being given by their difference.

8. Dual status of the group $SU(2)_L \otimes SU(2)_R$

In numerous works including the pioneer papers by Schwinger [7], Gürsey [8] and Touschek [9], which used the chiral symmetry group $SU(N)_L \otimes SU(N)_R$ or the group $U(1)_A$, it is difficult, if it's really possible, to find any commentaries on transformation properties of these group parameters in respect to the spatial reflection. As a rule it follows from the contents of the papers that the transformations of the group $SU(N)_L \otimes SU(N)_R$ or the group $U(1)_A$ do not violate P -symmetry. This rule, however, is broken in the left-right symmetric model of electroweak interactions [10-12] and in the unified models of strong and electroweak interactions including the first one as its element. Actually, in the papers [10-12] there are not at all indications that any component of one or another Higgs multiplet is the space pseudoscalar. We are forced to admit that all components of all Higgs multiplets, and, consequently, all parameters of the used group $SU(2)_L \otimes SU(2)_R$ are space scalars. Transformations of this group, generally speaking, violate the P -symmetry. If, for example, the initial state of a spinor field possesses a definite parity, then the state, obtained due to a transformation of the type (18), already does not possess it. The group in question itself does not establish any relation between coupling constants of two W -bosons to fermions. Therefore in the left-right symmetric model one introduces some discrete symmetry which transforms $SU(2)_L$ to $SU(2)_R$ but it is not identified with the spatial reflection. So one can state that the group $SU(2)_L \otimes SU(2)_R$ of this model is a secondary symmetry group produced by 2-multiple scalar isotriplet of the group $G = SU(2) \otimes L_+^\dagger \otimes (A \text{ discrete symmetry group})$, this discrete symmetry being not fully defined.

9. On P -properties of the physical vacuum and the gauge fields of electroweak interactions

Clarify now principal points related to the P -symmetry and its violation in electroweak interactions, if one corrects the left-right symmetric model so that it would be P -invariant before the spontaneous symmetry breaking. Note at once, that this correction does not affect values of the cross-sections and the decay probabilities.

Initial P -invariance and its observed violation is ensured by the local double symmetry produced by the representation $T = (\text{isotriplet}, \text{scalar}) \oplus (\text{isotriplet}, \text{pseudoscalar}) \oplus (\text{isosinglet}, \text{scalar}) \equiv (1, s) \oplus (1, p) \oplus (0, s)$ of the group $G = SU(2) \otimes L^\dagger$. Corresponding parameters of the secondary symmetry transformations and the gauge fields will be denoted as $\theta^{1s} = \{\theta_j^{1s}\}$, $\theta^{1p} = \{\theta_j^{1p}\}$, θ^{0s} ; $\mathbf{B}_\mu^{1V} = \{B_{j\mu}^{1V}\}$, $\mathbf{B}_\mu^{1A} = \{B_{j\mu}^{1A}\}$, B_μ^{0V} ($j = 1, 2, 3$; the indices V and A mean the polar and axial 4-vectors, respectively).

In the fermionic sector we restrict ourselves by isodoublet consisting of the fields of electronic neutrino ν_e and electron e . Its secondary symmetry transformations, as well as all others, are written in the form, explicitly satisfying the conditions of Definition 1

$$\psi' = \exp \left(-\frac{i}{2} \boldsymbol{\tau} \theta^{1s} - \frac{i}{2} \gamma^5 \boldsymbol{\tau} \theta^{1p} + \frac{i}{2} \theta^{0s} \right) \psi, \quad (21)$$

with $\psi^T = (\nu_e, e)$.

Checking that the transformations (21) constitute a group we introduce the above mentioned gauge fields with such phases that in the Lagrangian of their interaction with leptonic field ψ

$$\mathcal{L}_{int} = -\frac{1}{2\sqrt{2}} \bar{\psi} \left(g_{1V} \gamma^\mu \boldsymbol{\tau} \mathbf{B}_\mu^{1V} + g_{1A} \gamma^\mu \gamma^5 \boldsymbol{\tau} \mathbf{B}_\mu^{1A} - g_0 \gamma^\mu B_\mu^{0V} \right) \psi \quad (22)$$

the coupling constants g_{1V} , g_{1A} and g_0 are positive. The formulae (21) and (22) give also a knowledge of covariant derivatives corresponding those or others gauge transformations of the secondary symmetry group.

The most general form of the global secondary symmetry transformations of the fields \mathbf{B}_μ^{1V} and \mathbf{B}_μ^{1A} is

$$\begin{pmatrix} \mathbf{B}_\mu^{1V} \\ \mathbf{B}_\mu^{1A} \end{pmatrix}' = \exp \left[-i \begin{pmatrix} a_1 \mathbf{t} & 0 \\ 0 & a_2 \mathbf{t} \end{pmatrix} \boldsymbol{\theta}^{1s} - i \begin{pmatrix} 0 & b_1 \mathbf{t} \\ b_2 \mathbf{t} & 0 \end{pmatrix} \boldsymbol{\theta}^{1p} \right] \begin{pmatrix} \mathbf{B}_\mu^{1V} \\ \mathbf{B}_\mu^{1A} \end{pmatrix}, \quad (23)$$

where $\mathbf{t} = \{t_j\}$ ($j = 1, 2, 3$) are the generators of adjoint representation of the group $SU(2)$, and a_1 , a_2 , b_1 and b_2 are arbitrary constants. A requirement the transformation (23) to be orthogonal and the Lagrangian (22) to be invariant under the global transformations (21) and (23) is fulfilled if and only if $a_1 = a_2 = b_1 = b_2 = 1$ and $g_{1V} = g_{1A} \equiv g$.

We introduce the Higgs field Φ consisting of scalar $\phi^{\frac{1}{2}s}$ and pseudoscalar $\phi^{\frac{1}{2}p}$ isodoublets and taking some vacuum expectation values of its neutral components with isospin projection $-1/2$: $\langle \phi_{-1/2}^{\frac{1}{2}s} \rangle = v_s$, $\langle \phi_{-1/2}^{\frac{1}{2}p} \rangle = v_p$. Relative phases of the fields $\phi^{\frac{1}{2}s}$ and $\phi^{\frac{1}{2}p}$ are fixed so that the secondary symmetry transformations have the form

$$\begin{pmatrix} \phi^{\frac{1}{2}s} \\ \phi^{\frac{1}{2}p} \end{pmatrix}' = \exp \left[-\frac{i}{2} \begin{pmatrix} \boldsymbol{\tau} & 0 \\ 0 & \boldsymbol{\tau} \end{pmatrix} \boldsymbol{\theta}^{1s} - \frac{i}{2} \begin{pmatrix} 0 & \boldsymbol{\tau} \\ \boldsymbol{\tau} & 0 \end{pmatrix} \boldsymbol{\theta}^{1p} - \frac{i}{2} \boldsymbol{\theta}^{0s} \right] \begin{pmatrix} \phi^{\frac{1}{2}s} \\ \phi^{\frac{1}{2}p} \end{pmatrix}. \quad (24)$$

The Higgs fields $\phi^{\frac{1}{2}s}$ and $\phi^{\frac{1}{2}p}$ are sufficient to reproduce all results of the Weinberg-Salam model in the region of existing energies, except for the generation of fermionic masses. It's just needed that the relation $|v_s - v_p| \ll |v_s + v_p|$ would be fulfilled and the vacuum expectation value of any other field of Higgs's type would be much less than $|v_s - v_p|$. Concerning fermionic masses one can suggest up to some moment that they appear as a result of interaction of fermions with Higgs fields constituting the multiplets $\{\phi^{1s}, \phi^{0p}\}$ and $\{\phi^{1p}, \phi^{0s}\}$, whose transformations are generated by the elements of the type (19-20).

In further formulae we neglect the vacuum expectation values of all fields apart from $\phi^{\frac{1}{2}s}$ and $\phi^{\frac{1}{2}p}$. From the Lagrangian $\mathcal{L}_\Phi = |\mathcal{D}_\mu \Phi|^2$, where \mathcal{D}_μ is a covariant derivative corresponding to the gauge transformations (24), we get that the electromagnetic field A_μ , the fields of light $W_\mu^{(1)\pm}$, $Z_\mu^{(1)}$ and heavy $W_\mu^{(2)\pm}$, $Z_\mu^{(2)}$ intermediate bosons are described by the following relations

$$A_\mu = \frac{1}{\sqrt{g^2 + g_0^2}} (g_0 B_{3\mu}^{1V} + g B_\mu^{0V}), \quad (25)$$

$$W_\mu^{(i)\pm} = \frac{1}{2} [(B_{1\mu}^{1V} - \eta_i B_{1\mu}^{1A}) \mp i (B_{2\mu}^{1V} - \eta_i B_{2\mu}^{1A})], \quad (26)$$

$$Z_\mu^{(i)} = \frac{\alpha_i}{\sqrt{g^2 + g_0^2}} (g B_{3\mu}^{1V} - g_0 B_\mu^{0V}) - \beta_i B_{3\mu}^{1A}, \quad (27)$$

$$m_{W^{(i)}}^2 = \frac{g^2}{4} |v_s - \eta_i v_p|^2, \quad (28)$$

$$m_{Z^{(i)}}^2 = \frac{|v_s|^2 + |v_p|^2}{4} \left(g^2 + \frac{g_0^2}{2} - \eta_i \frac{g_0^2 \sqrt{\gamma^2 + 1}}{2\gamma} \right), \quad (29)$$

where

$$\eta_1 = 1, \quad \eta_2 = -1, \quad \gamma = \frac{g_0^2 (|v_s|^2 + |v_p|^2)}{2g \sqrt{g^2 + g_0^2} (v_s v_p^* + v_s^* v_p)},$$

$$\alpha_i = \sqrt{\frac{1}{2} \left(1 - \eta_i \frac{\gamma}{\sqrt{\gamma^2 + 1}} \right)}, \quad \beta_i = \eta_i \sqrt{\frac{1}{2} \left(1 + \eta_i \frac{\gamma}{\sqrt{\gamma^2 + 1}} \right)}. \quad (30)$$

From Eqs. (22) and (25-27) we get

$$\begin{aligned} \mathcal{L}_{int} = & e_0 \bar{e} \gamma^\mu e A_\mu - \frac{1}{2\sqrt{2}} \sum_{i=1}^2 [g \bar{\nu}_e \gamma^\mu (1 - \eta_i \gamma^5) e W_\mu^{(i)+} + \text{H.c.} \\ & + \bar{\nu}_e (\alpha_i \sqrt{g^2 + g_0^2} \gamma^\mu - \beta_i g \gamma^\mu \gamma^5) \nu_e Z_\mu^{(i)} + \bar{e} (-\alpha_i \frac{g^2 - g_0^2}{\sqrt{g^2 + g_0^2}} \gamma^\mu + \beta_i g \gamma^\mu \gamma^5) e Z_\mu^{(i)}]. \end{aligned} \quad (31)$$

It follows from Eqs. (28) and (31) that the observed domination of the left-hand weak charged current is possible if and only if $v_s \neq 0$, $v_p \neq 0$ and $\arg(v_p/v_s) \neq \pm\pi/2$. This means that **the physical vacuum does not possess of a definite P -parity** because $Pv_s = v_s$, $Pv_p = -v_p$. This is the first principal point which has not been clear in the left-right symmetric model. It's interesting, that at the time when the spontaneous breaking of the gauge symmetry and a related to it understanding of physical vacuum was not been formulated yet, Nambu and Jona-Lasinio noted in their paper [13], "that the γ^5 transformation changes the parity of the vacuum which will be in general a superposition of states of opposite parities".

It follows from Eqs. (26-27) that **the fields of all intermediate bosons constitute a superposition of polar and axial 4-vectors, these vectors having an equal weight in the fields of W -bosons**. This is just the second principal point which has not been clear in the left-right symmetric model, though, on our opinion, it has the same powerful significance as the form of weak currents.

If we anywhere considered the intermediate boson masses and the coupling constants of Z -bosons to fermions as functions of the field values v_s and v_p , then we would get from Eqs. (26)-(31) that under the spatial reflection the $Z^{(1)}$ -boson coupling constants turn into the $Z^{(2)}$ -boson coupling constants and vice versa, as well as

$$Pm_{W^{(1)}}^2 = m_{W^{(2)}}^2, \quad Pm_{Z^{(1)}}^2 = m_{Z^{(2)}}^2, \quad PW_\mu^{(1)\pm} = (-1)^{\delta_\mu} W_\mu^{(2)\pm}, \quad PZ_\mu^{(1)\pm} = (-1)^{\delta_\mu} Z_\mu^{(2)\pm}, \quad (32)$$

where $\delta_\mu = 0$ if $\mu = 0$, and $\delta_\mu = 1$ if $\mu = 1, 2, 3$. This would cause an invariance of the Lagrangian (31).

Thus, the corrected version of the left-right symmetric model of electroweak interactions leads to the logically completed interpretation of the P -invariance violation, and reveals a full analogy between transformation properties in respect to the spatial reflection of weak currents and corresponding gauge fields of the intermediate bosons.

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